Deficit Comparison test validating Reservoir Reliability

Setting the stage

When performing reanalysis of the reliability of the reservoirs in the Orange basin using the extra 10 years of data up to 2004, the simulations based on the freshly fitted time series give draw-down plots in which some of the deficits are shallower than previously estimated, possibly giving a false sense of reliability. Furthermore, the addition of the ten years of data has caused a significant change in the choice of time series model in many cases.

To determine the reasons for this apparent paradox, it was decided to look at the problem from the point of view of the flow records and what characterises them. In addition, a novel method of time series model verification called Deficit Comparison was devised. This involves feeding the chosen reservoir over a ten year period with many (1001) simulations of possible flows and subjecting the reservoir to different levels of withdrawal (50%, 70% and 90% MAR), starting the reservoir full at the time origin. The method used was the 'sudden' withdrawal method defined by Pegram (1980) which, on an annual basis, ensures that the drawdown sequence is below the monthly trajectories. These 1001 levels at each forecast time step are ranked and their quantiles plotted to replicate the standard storage deficit plots produced by WRP. Then the historical records (75 or 85 years, depending on the perspective) are progressively divided into 10 year sequences starting at successive years through the record (1920 to 1929, 1921 to 1930, …) and fed into the same 'reservoir' starting full, with matching withdrawal protocol. The historical deficit plotting was limited to points so as not to confuse the plots with many trajectories. A sample plot of the Deficit Comparison test for Katse, fed by the earlier 75-year sequence follows. The time series model used here was the one used since 1997: an MA(1) model with parameter -0.2415.

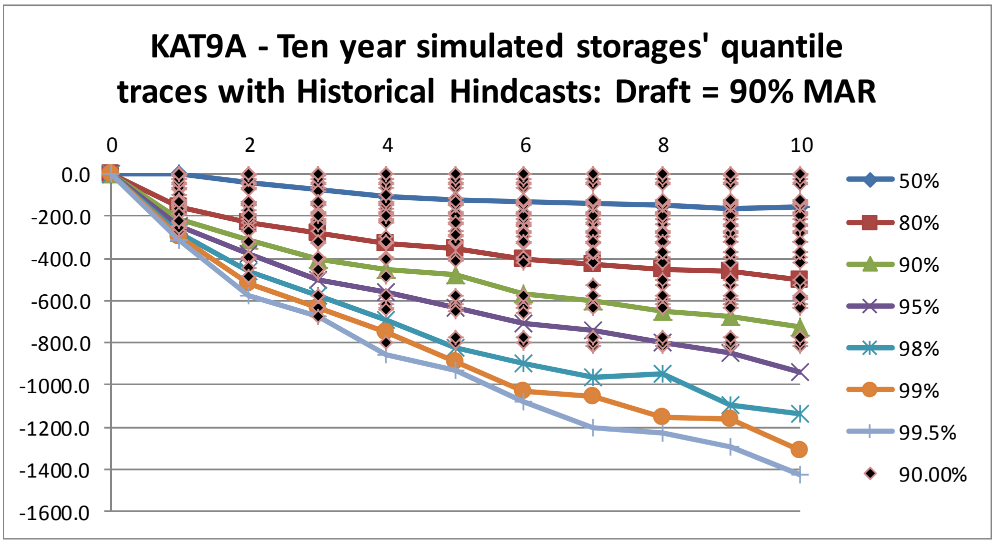

Figure 1 Deficit Comparison test for Katse, fed by the 75-year sequence [1920-1994]

Here, as in the following, the analysis is confined to Katse (other reservoirs' responses are available in the accompanying spreadsheet called MODEL_SELECTION.xlsx). For an 85-year sequence, one would expect the maximum historical deficit at any lead time to have a recurrence interval (RI) of about 150 years, based on the Gringorten plotting position: RI = $(n + 0.12) / (k - 0.44)$. Therefore, theoretically, the lowest historical point should lie below the 99% line, but not fall below the 99.5% (200 year) line. The diagrams like Figure 1 based on the 75 year period ending in 1994 (lowest historical deficit with RI = 134 years) do not show any violations. In contrast, the plots of Katse for the second 85 year period ending in 2004, show many. An example for the longer period, using the model selected as best by the BLEND criterion incorporating the AIC and the Drift, now part of the cleaned up code in AnnualBLEND.FOR, is shown in Figure 2. Here the lowest historical deficit in forecast year 4 lies well below the 99.5% line, which is a worry. Just for information, the red marker indicates the lowest of all historical deficits and lies where expected.

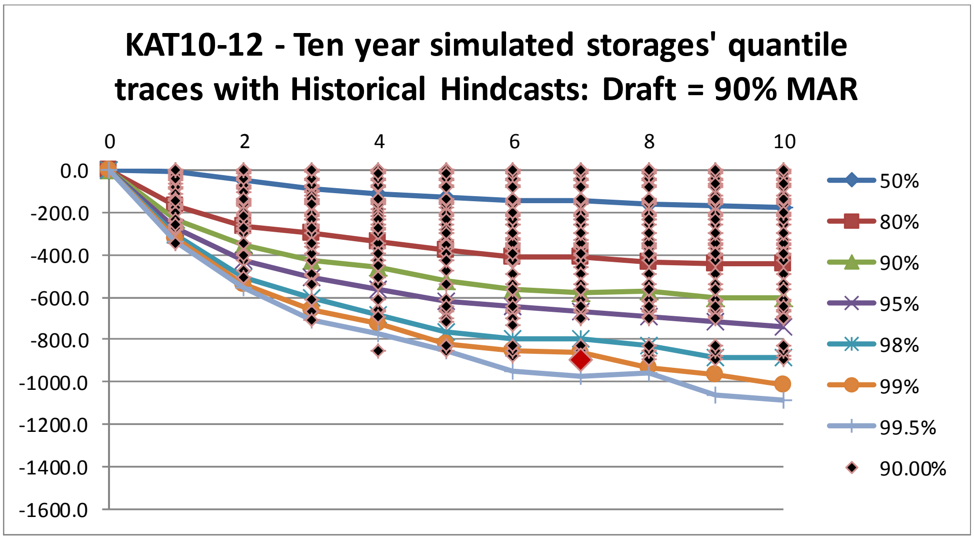

Figure 2. Deficit Comparison test for Katse, fed by the 85-year sequence [1920-2004]. ARMA(1,2) model selected as best by the BLEND criterion incorporating the AIC and the Drift.

This drop below the 1:200 year line, in all the Katse plots associated with the longer 85-year period, occurs regardless of whether one views the plots of the 'best' selected time series, or whether we use all of the fitted models, as seen in the spreadsheet. Not only was the quantile threshold violated by the best time series model (as fitted by the AICc) but it was also crossed by those ARMA models selected by the hybrid criterion BLEND using a weighting of AIC and DIST. The latter (DIST) measure is based on matching the extreme historical and simulated deficits, which has been working for 16 years since the last serious modification of the ANNUAL.FOR code since 1996.

Given the above, it was decided to do a more searching analysis for the cause of the paradox. The following images in Figure 3 show what are thought to be the cause of the change in reservoir drawdown response due to the 10 years additional data of the historical time series.

To start from a theoretical point of view, the reliability of a reservoir under a given withdrawal regime is inversely proportional to the coefficient of variation (Cv) of the time series feeding the reservoir (Pegram, 1980). In the case of Katse, four diagrams are included in Figure 3: (a) the recorded annual flows' 5-year means, (b) their 5-year standard deviations (std), (c) their 5-year Cvs, all over-plotted by their 15-year moving averages to pick up the trend, followed by (d) the trace of the time series of deficits for three levels of withdrawal.

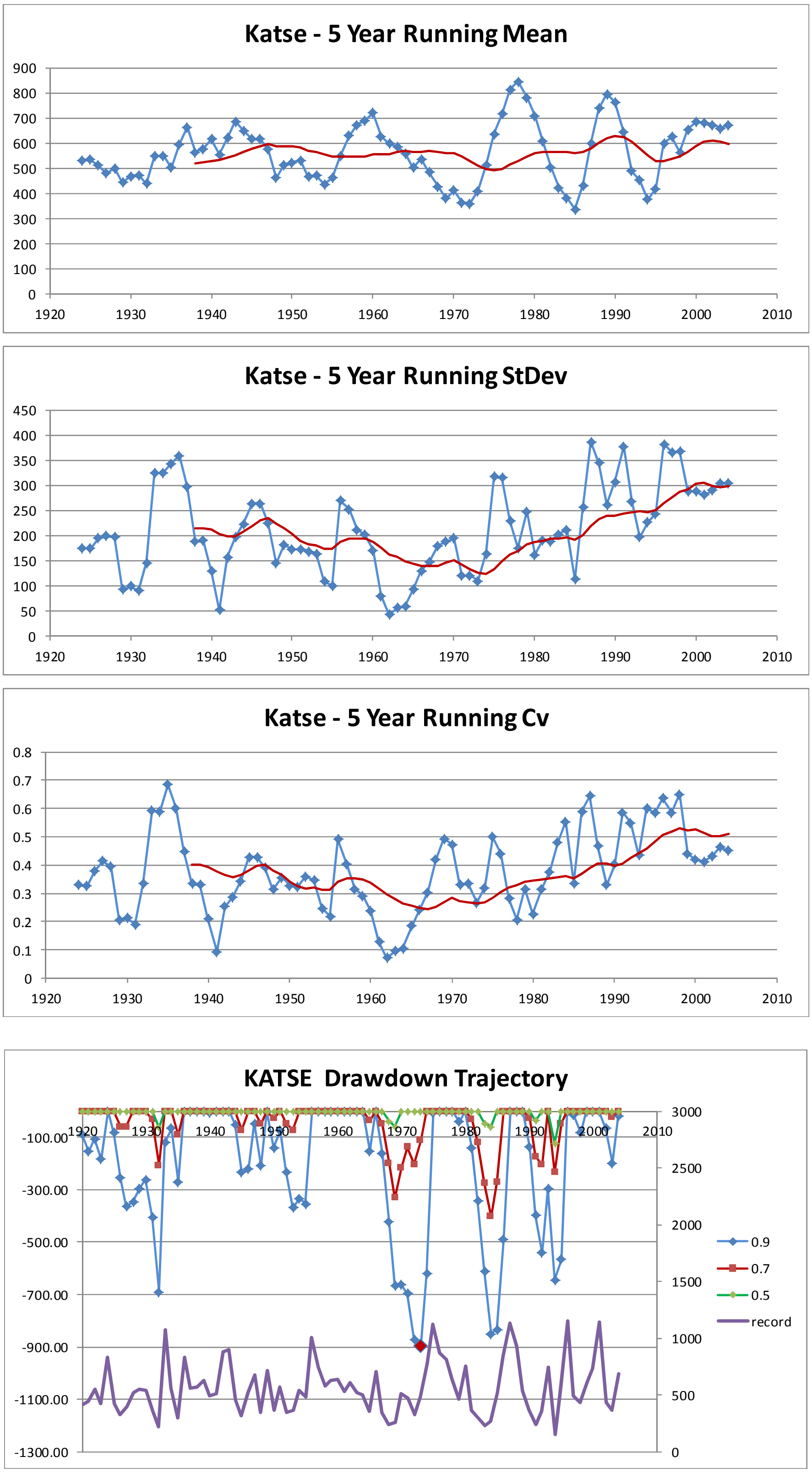

Figure 3: Katse dam-site's (a) recorded annual flows' 5-year means, (b) their 5-year standard deviations (std), (c) their 5-year Cvs, all over-plotted by their 15-year moving averages to pick up the trend, followed by (d) the trace of the time series of deficits for three levels of withdrawal, with the original annual time series coaxially plotted – scale to the right. The red marker shows the lowest deficit, echoing that in Figure 2.

What is clear from Figure 3, is that the means are quite stable over time (I have not done any comparative statistical tests like Mann-Whitney), however, from the mid-1970s onwards the variability has increased markedly as shown by the 5-year averages of the stds. Also obvious is that the range between high and low flows has increased since the 1970s as can be seen in the bottom panel (d). Putting aside the possibility that a large part of the Katse record has been infilled, the time series is evidently (i) not stationary in time and (ii) that there has been material change in the character of the flows in the last 24 years. This begs the question as to whether the changes are artificial or natural (incidentally, since what year has there been a reliable stream-gauge at the site of the dam?)

As a test, the 'best' estimated time series fitted by ANNUAL to the 85-year Katse record and their simulations were used in the Deficit Comparison test. The following 3 figures show the poor performance of the models in capturing the correct depth of historical deficit at all drafts. It should therefore NOT be used in analysis of the system. The 'best' time series gives one of the worst Deficit Comparison tests.

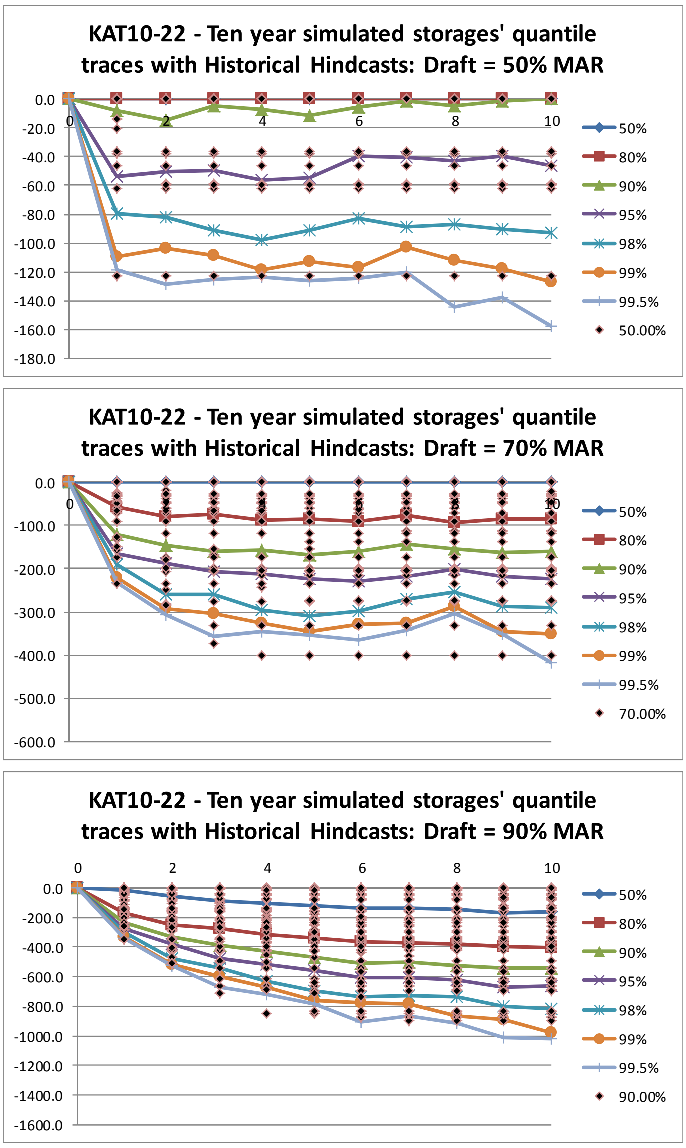

Figure 4 Katse Deficit Comparison test for the period 1920-2004 using the 'best' ARMA(2,2) model, revealing its inadequacy in a storage-yield context in this case.

Mindful of the observed lack of stationarity in the Katse historical sequence (especially the Cv) the last 50 years of flow were selected for modelling as a comparator. Interestingly, the first serial correlation coefficient (scc) of the sequence jumped from 0.228 for the 85-year record to 0.364 for the last 50-years. The comparative scc plots in Figure 5 show that the correlation at all lags has materially increased in the latter period.

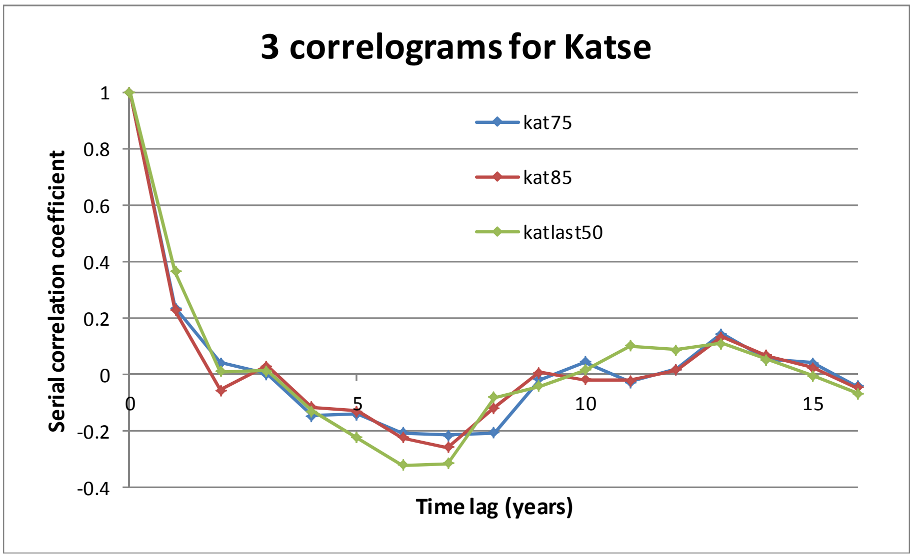

Figure 5 Comparative plots of serial correlation coefficients fitted to three segments of the Katse time series: 75-year [1920-1994], 85-year [1920-2004] and 50-year [1955-2004]

Evident in all the scc plots of Figure 5 is quasi-periodic behaviour with a length of about 13 years, and a shape that is classically fitted by either an ARMA(2,0), ARMA(2,1) of even an ARMA(2,2) time series model. In any event, the model chosen by the modified code [AnnualBLEND.FOR] was an ARMA(1,2) and it produced the Deficit Comparison diagram between the simulated quantiles and the historical sequence of withdrawals in Figure 6 following. Note that the mean of the 50 years is a little higher than that for the whole 85-year period. This test validates the model choice in this case.

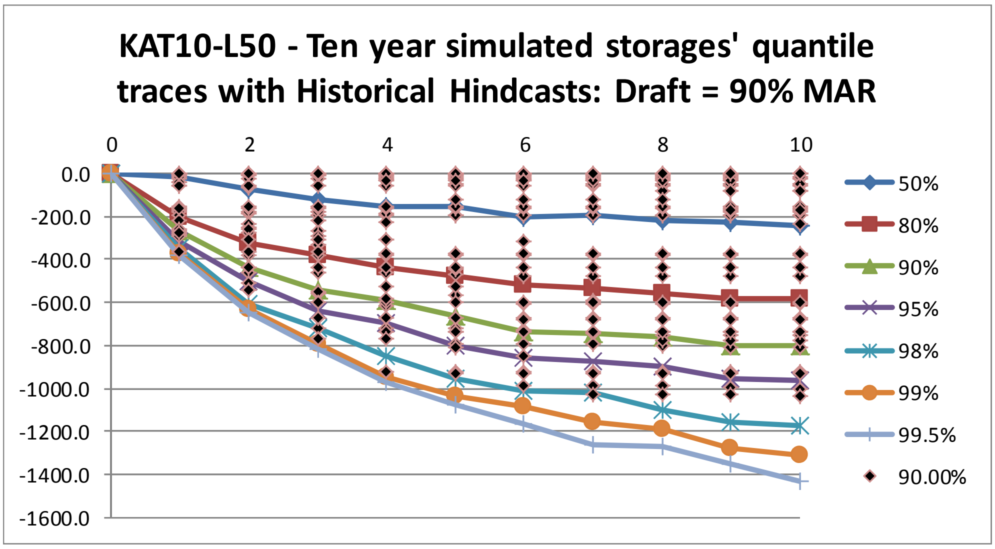

Figure 6 Deficit Comparison diagram comparing the simulated quantiles and the historical sequence of withdrawals using the model chosen [ARMA(1,2)] by the modified code [AnnualBLEND.FOR]

Summary:

The conclusions emerging from these explorations include:

- There has been a significant change in the character of the Katse time series in the latter part of the record compared to the earlier part.
- Modelling the time series as if it is stationary (which is the assumption of the current programs) may need to be questioned

Thus, a suggestion for the adaptation of the models is to use a progressive, or evolutionary, time series modelling technique giving more weight to the recent, compared to the earlier record.

A methodology with which I am familiar which can achieve this end, and which I have used in other studies, is called Recursive Least Squares time series modelling. It incorporates the adaptive skill of a Kalman Filter and can be tuned to treat the whole time series either equally (as above) or using exponential weighting of the more recent past, to any desired degree. Thus the record as a whole would contribute to the time series modelling process, but the method allows for recognising the non-stationarity of the observed record if desired and giving more weight to recent observations. I think this work would be a first in the reservoir operating world and the tool would allow us to sensibly adapt to whatever gradual or sudden changes we encounter as our recorded data advances into the uncertain future.

Recommendation:

Setting that idea aside for the time being, my recommendation is to model the currently available 85 year record with the same type(s) of time series model(s) as were chose by the program ANNUAL to model the earlier 75-year record, with the proviso that the newly computed parameters be used. If there is any doubt about the suitability of such a chosen time series model to provide the right level of assurance, the choice could be assisted by using the Deficit Comparison technique/test. The code for this analysis is in the program TRACES.FOR and has already been offered to WRP consulting under this mini-contract. My only claim is that I maintain the intellectual property rights on this idea, so that I may publish under my name if the spirit moves me.

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29 November 2012

Reference

Pegram, G.G.S., (1980). On Reservoir Reliability. *Journal of Hydrology*, Vol.47, pp.269-296.